

CHAPTER 6

BLOCKMODELING OF VALUED NETWORKS

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6.1 Introduction

In the wide variety of networks that connect and make up our worlds, relations not only either exist or not, but often carry a weight. Friendships can be ranked, interactions can be timed, and economic exchanges can be valued, details that provide us with a deeper, nuanced and higher-resolution understanding of such networks than is provided by the mere existence of ties.

As with most methods and heuristics in network analysis, approaches for clustering¹ or blockmodeling are primarily geared to binary data [277, p. 127; 279]. For the set of indirect methods that do work with valued networks when determining clusters of equivalent actors, valued data still pose a dilemma when interpreting possible blockmodels derived from such approaches. Since the ideal blocks² of generalized blockmodeling (as well as density-based structural blockmodeling) are specified in terms of binary ties, these are not readily

¹Following the terminology in this book, the terms “cluster” and “clustering” as used in this chapter refer to subsets of, and the procedure of identifying, actors that fulfill some meaningful definition of equivalence (also referred to a “position” in some literature).

²In this chapter, “block” refers to the non-overlapping submatrices in a blockmodel as delineated by one or two clusters (i.e. subsets of actors; see the previous footnote).

comparable with the intra- and inter-block patterns of valued relations. Valued networks are therefore often dichotomized, either prior to identifying equivalent sets of actors or when determining patterns within and between clusters using a statistically, theoretically or arbitrarily determined network-wide threshold.

Whereas dichotomization can be theoretically motivated in some cases, *particularly when it comes to ranked/ordinal data*, dichotomization constitutes an inevitable reduction of the resolution and detailed level of valued data. In addition, the setting of a global cut-off to distinguish prominent and non-prominent valued ties is inherently problematic when actors in a valued network have different relational capacities [258, 295]. Good examples of networks in which actors have different relational capacities are trade networks. For instance, whereas the USD 59.4 billion worth of commodities that went from Germany to Austria in 2010 represents half of Austria's total imports, this very same flow only corresponds to 8 percent of Germany's total exports.³ With unequal relational capacities such as between Austria and Germany, where the local perception of what constitutes a prominent tie differs widely, the use of absolute (network-wide) dichotomization thresholds show us, at best, just one possible way of looking at the data. In contrast to the trade flow example, an assumption of equal relational capacities would be more reasonable when, for instance, tracking the interaction times of school children during a common 45-minute break.

Besides dichotomization, other types of transformation can be useful for analyzing valued networks, both for identifying clusters and discerning how the block patterns of valued ties relate to the set of ideal binary blocks used in blockmodeling. A growing number of direct and indirect blockmodeling heuristics is also designed explicitly for valued networks, approaches that often are combined with, or based on, various transformations.

This chapter provides an overview of approaches, classical as well as recent innovations, to the clustering/blockmodeling of valued networks. This chapter focuses exclusively on one-mode, single-relational, non-signed⁴, possibly directional and possibly with self-ties, valued networks and the particularities that apply to the clustering/blockmodeling of such networks. We thus do not delve deeper into the properties of networks and methods that are equally relevant to binary networks⁵. We also do not cover stochastic blockmodeling, although versions of it for valued networks exist [e.g. 255].

In addition to the various direct and indirect methods applicable to valued networks, we describe various transformations found useful when clustering valued networks. The technicalities of each method and transformation are described, and we also discuss the particular variety of equivalence a respective method is geared to capture. We exemplify how different heuristics yield different results using two datasets, embodying different properties of valued networks: the EIES friendship data⁶ at time point 2 [282], and intra-European commodity trade [295].

The next section describes the different types of valued network data that exist. This is followed by a description of various transformations that can be applied to such data prior

³Taken from the EU/EFTA trade example in the second part of this chapter, we occasionally use this data in our theoretical section to exemplify the effects of transformations.

⁴Some methods described in this chapter are also appropriate for signed networks (e.g. homogeneity blockmodeling [279]).

⁵Such as hierarchical clustering methods and their different varieties: although useful for grouping actors based on indirect measures of equivalence, such tools are equally useful for binary networks as well. That said, we do however demonstrate that different hierarchical clustering methods yield different partitions in our first EIES data example.

⁶EIES data obtained from <https://sites.google.com/site/ucinetsoftware/datasets/freemanseiesdata>

to clustering methods being used. An overview of clustering methods and heuristics for valued networks follows, divided into indirect and direct approaches. An example section comes next in which we demonstrate various clustering/blockmodeling techniques on our two example networks. A concluding section rounds off the chapter.

6.2 Valued data types

As with statistical variables generally, tie values⁷ can be measured on different scales. In this chapter, only networks where ties can be treated as measured on at least an interval scale are considered. We remain relatively liberal in terms of what can be treated as interval scales, as is often the practice in the social sciences. Therefore, tie values on 1–5 rating scales (Likert-type items) and similar are deemed acceptable.

Below, we list some of the more commonly used types of valued networks based on the meaning and/or origin of ties:

- A subjective judgment of relationships using different rating scales (e.g. a 5-point scale with descriptions at the extremes only, a 5-point scale with a description of each category, line measurement . . .). The quality of measurement using different scales has been discussed elsewhere [e.g. 281, 284]. These rating scales share certain features:
 - The measurement is subjective (based on the perception of the responding actor).
 - At least the scale extremes are defined and are the same for all actors. In principle at least, all actors can also select the extreme option. This also means the definition of a "strong" tie is given from the outside (but interpreted by the respondent). Examples of such networks include, among others, all kinds of perceived social support networks, advice networks [e.g. 302] and friendship networks [282].
- Direct measurements of different flows or the frequency of some kind of interactions. Examples of such networks are email exchanges between people [301, 309], citations among journals [257] and trade among countries [295]. While these measurements may lack a pre-specified maximum, natural limits may exist such as when mapping minutes of school kids' interaction during a lunch break of limited duration. Typically, however, the perception of what constitutes a strong tie in such measurements varies among actors.

One distinction we concentrate on in this chapter is whether actors have equal or unequal relational capacities [295]. That is, whether all actors, at least in theory or principle, have similar capabilities for creating/maintaining ties of a certain strength and/or having a similar upper ceiling with respect to the sum of such tie strengths. This is very closely related to the question of how to determine which ties can be deemed important, strong, prominent etc. Can we determine this uniformly for the whole network, for parts of the network, or should this be done per actor or even at the dyadic level?

In more practical terms, when doing blockmodeling on networks where at least empirically different relational capabilities exist (e.g. a notable variance in valued in- and out-degrees), two interrelated questions must be addressed prior to the analysis:

⁷Also known as tie weights, particularly in the network science literature.

- Does the perception of what constitutes a strong tie differ significantly among actors? Put differently: do relational capacities differ among actors?
- If so, do we want such differences in relational capacities to influence our results? Are such would-be differences part of what we are looking for, or do we want to somehow discount such differences in our analysis?

Depending on the response to such questions, a variety of transformations and metrics exist that, with different variations in the specific conceptualization of equivalence, can be used in the context of valued blockmodeling.

6.3 Transformations

Prior to blockmodeling valued networks, relational data are often transformed in various ways to either adapt them to methods and heuristics that are primarily used for binary data or to assist a method in assessing a specific kind of tie prominence. The most trivial transformation is that of dichotomization: using a network-wide absolute threshold, the valued network is binarized to which conventional heuristics and methods apply. Treating dichotomization separately in what follows, other types of transformations have been used to cluster/blockmodel valued data. These can be separated into element-wise transformations where the individual ties are transformed independently of each other, or what we may call structural transformations where the transformation of a valued tie in some way depends on the properties of other valued relations in the network.

Transformations of valued networks can be combined and applied at different stages of a clustering/blockmodeling analysis of valued networks. Describing the various steps and procedural pathways for going from the raw valued network to clusters of actors, we propose the schematic in Figure 6.1, mainly to show how the different steps described later in the chapter come together. In Section 6.6 we provide some guidelines (also in the form of a flow chart in Figure 6.2) for selecting suitable approaches (including transformation).

We thus identify four main procedures for identifying clusters in valued networks:

- Raw valued data → dichotomization → binary clustering method → clusters
- Raw valued data → (non-dichotomous) transformation → dichotomization → binary clustering method → clusters
- Raw valued data → (non-dichotomous) transformation → valued clustering method → clusters
- Raw valued data → valued clustering method → clusters

Once the clusters are defined and a blockmodel is created, the common final step (not included above) is to interpret the blockmodel and the inter- and intra-block patterns of ties. Such block and blockmodel interpretations can either be based on the original valued data, the would-be transformed data prior to clustering, or by an additional post-clustering transformation.

The choice of transformations to apply to valued data before and possibly after clustering might at times seem like a somewhat arbitrary decision, often based on what yields the most intuitive, plausible, and interpretable results. Yet each transformation can be seen as the operationalization of a particular interpretation of prominence in the context of the

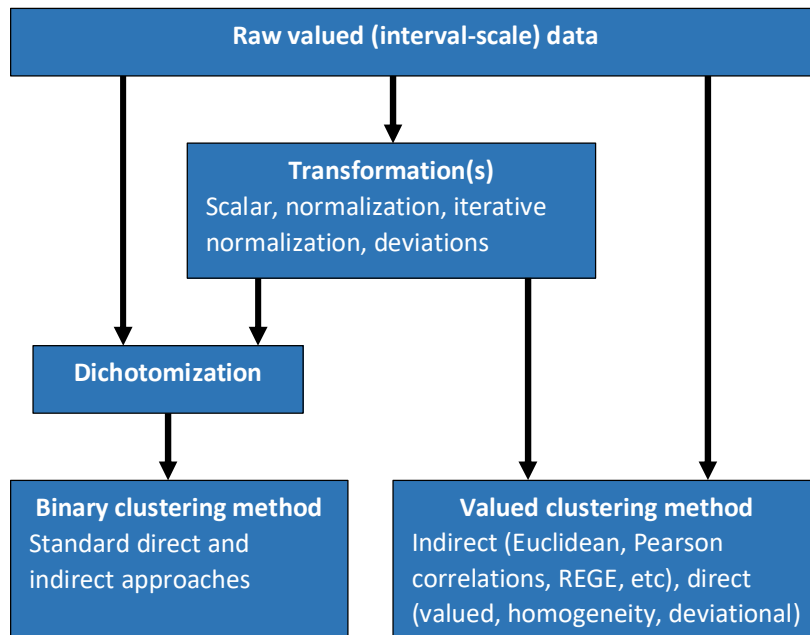


Figure 6.1: Procedures for clustering valued data

valued network being studied. An example of this interplay between different transformations and final interpretations is found in Breiger’s study [269] of intra-OECD trade. When applying the CONCOR algorithm (see below) to the same valued network of trade flows, but testing two different pre-CONCOR transformations – a dichotomization that keeps the quintile of the largest ties, and an iterative row-column normalization procedure – Breiger demonstrates how final interpretations of a dataset are intrinsically tied to the transformation methods that are chosen. We show a similar effect in our EFTA trade example.

In what follows, we look at various transformations that have been applied in the valued blockmodeling context. Starting off with the element-wise scaling transformations and dichotomization, we then consider normalization-based transformations and other structural transformations.

6.3.1 Scaling transformations

Apart from absolute-value dichotomization, various element-wise scaling transformations can be applied prior to an analysis, transformations that thus retain the rank order of the valued ties. An example is the world-system-inspired study of Mahutga [288] in which he analyzes international trade flows of various commodity bundles for five points in time between 1965 and 2000, i.e. valued networks whose unequal relational capacities among countries (e.g. as given by the skewness of the tie value distributions) is a characteristic feature of the network. Determining regular equivalence using the REGE algorithm (see below), Mahutga applied a log-10 transformation of the commodity flow matrix prior to the algorithm that “maintains the relative differences between countries while aiding the al-

gorithm” [288, p. 1884]. Such transformations are appropriate when the skewness or some other property of the tie value distribution is problematic in terms of the blockmodeling method later used, similar to e.g. using a log transform prior to regression analysis.

6.3.2 Dichotomization

The most common transformation of valued data is to dichotomize them. Irrespective of when the dichotomization is performed (see Figure 6.1), its feasibility hinges on the substantive research question and the properties of the valued ties, particularly their distribution. For ranked/ordinal data, where each value represents a particular strength, e.g. the quality or intensity of a social tie [e.g. 282, 305], dichotomization is used to filter out that particular strength of a tie. This is, we argue, the only case where dichotomization may be preferable to using valued data. If the range of continuous values is constrained, e.g. when mapping playtime among school kids during a 45-minute lunch break, it could be feasible to distinguish prominent (1) and not-so-prominent (0) pair-wise interaction on the basis of a system-wide absolute threshold. When theory offers no guidance on the suitability of such thresholds, distributions/histograms of tie values and valued degrees can be useful for determining possible thresholds. One can also try different dichotomization thresholds on the same valued network and compare the findings: as was done, e.g., in [276].

Rather than choosing a given absolute threshold for dichotomizing valued data, this can also be decided on the basis of total flow values. For instance, having sorted the valued ties in a network, one option is to retain a certain share of the largest values [269, p. 361], i.e. pruning away the corresponding share of the smallest values.

Dichotomization is not only directly applicable to raw valued data but can be, and often is, used as a secondary transformation prior to clustering. For instance, whereas normalization can be used to transform a valued network (see below), the normalized version of the network may have to be subsequently dichotomized prior to finding clusters using binary-oriented methods [see, e.g., 294]. As normalization and similar structural transformations are typically applied to counter skewness in valued degree distributions, i.e. as an attempt to counter unequal relational capacities among actors, choosing a dichotomization cut-off for the normalized version of the network should be easier than with the raw valued network.

6.3.3 Normalization procedures

Normalization is a structural transformation that rescales the individual dyads relative to the values of other ties. It can either be applied prior to identifying clusters to assist a particular clustering heuristic [e.g. 257, 269] or in the interpretation of blocks [e.g. 294]. Normalization can either be done on the basis of individual actors and their immediate ego-networks or for the entire network as a whole. The former type of normalization can either be done with respect to inbound (column-wise normalization) or outbound ties (row-wise normalization). Normalization is applicable to all types of valued networks, whether directional or symmetric, and can include self-ties.

The most common type is marginal (or sum) normalization where each value is rescaled such that the sum of all values equals a given constant (typically unity). A full-matrix marginal normalization thus implies a rescaling of all relational values so that they sum up to unity. Although this does not change the relative differences between values and thus retains would-be unequal relational capacities, full-matrix marginal normalization is a useful transformation for comparing two or more networks as it rescales the values of each

network into a common standardized value range. For instance, when comparing pupil interactions during a 45-minute break with those of a 1-hour break, full-matrix normalizations of both could allow for an easier comparison of the two.

Row- and column-wise marginal normalization are transformations where, respectively, the out- and in-degrees for each actor sum up to unity.⁸ In the above example on Germany's exports to Austria, a row-wise marginal normalization would thus transform this dyad to 0.078 whereas a column-wise marginal normalization would transform it to 0.504. In these directional normalizations, the prominence of a tie is thus determined on a directional per-actor basis, indicating the share of a tie value with respect to the total (in- or outbound) tie values. A marginal normalization on rows (columns) is calculated by dividing each valued relation with the total valued outdegree (indegree). If an actor has a zero out- or in-degree, its corresponding row- or column-normalized values are in principle undefined, but often coded as 0.

For directional valued networks, assuming a lack of theoretically compelling reasons to only look at outbound or inbound ties, another possibility is to combine the two marginal-normalized matrices for rows and columns. In [294], given particular clusterings, both the row- and the column-wise normalized matrices were analyzed simultaneously to determine occurrences of regular blocks. Using a standardized threshold value, the row-regularity of blocks was identified by looking at the row-wise normalized matrix while the corresponding column-regularity was identified using the column-wise ditto. An alternative is to combine the row- and column-normalized matrices into a common matrix. Adding the elements of the row- and column-wise marginal-normalized matrices produces a matrix in which the cell values, ranging from 0 to 2, indicate the total relative significance of a valued flow for both actors. In the context of international trade, this composite metric has been used as a measure of the economic importance of individual flows [275, p. 182]. In the case of the directional trade flow from Germany to Austria (see above), this flow would then be transformed to 0.582.

Common alternatives to the marginal type of normalization are normalizations with respect to the mean, maximum, Z score and standard deviation, each applicable to the whole network as well as the ego-centric normalizations with respect to in- and outbound ties, with or without self-ties. Of these, the mean variety has an advantage in that it works in row- or column-normalizations where actors might lack out- or inbound ties, whereas the marginal-, standard deviation- and Z-score-based normalizations yield invalid results when an actor's corresponding in- or outdegree is zero.

6.3.4 Iterative row-column normalization

An extension of the above normalizations is to use the iterative row-column normalization approach [see, e.g., 300, p. 267; 296, 297, 291]. Repeatedly normalizing the valued data by rows, columns, rows, columns etc. will for most non-trivial data lead to a convergence where the sums of both the row and column vectors, respectively, sum up to unity (for marginal-normalization) or zero (for mean-normalization). The latter approach was used as a pre-clustering transformation by Breiger [269] and Baker [257]: whereas Breiger prepared his intra-OECD trade data prior to applying the CONCOR algorithm, Baker used this iterative approach prior to the Euclidean-based indirect measure of structural equivalence (see the next section).

⁸For symmetric (non-directional) valued networks, the row-wise marginal-normalized matrix is the transpose of the column-wise one.

The choice between the marginal- and mean-type iterative normalization is not arbitrary; instead, they are distinctly different transformations. The choice should primarily be made on substantive grounds. While the results of marginal row-column normalization can be thought of as values equivalent to raw ones in a hypothetical case where all units have exactly the same relational capacity, the meaning of “equivalent” for mean-based normalization is harder to assess. In addition to the mean and marginal iterative normalizations, similar iterative convergences occur when conducting row/column normalizations based on standard deviations (dividing each element in the row/column by the standard deviation of that row/column) or Z-scores (subtracting the mean from each element, subsequently dividing by the standard deviation). However, contrary to what is the case for the mean and marginal varieties where it is irrelevant whether the iteration starts with the rows or a column, that is not the case for the standard deviation and Z-scores iterative approaches.⁹

6.3.5 Transaction-flow and deviational transformations

The dilemma of unequal relational capacities precedes contemporary network analysis and so did the suggested solutions for distinguishing the prominent ties of a valued network from those that could be deemed less prominent. Much of this work was in the context of international trade and the much-debated transaction flow model and Relative Acceptance index of Savage and Deutsch [299; also see 285, 283, 256]. Savage and Deutsch aimed to create a null (quasi-independence) model of flow value probabilities that accounts for the individual distributions and magnitudes of both imports and exports of each country¹⁰. Contrasting these probabilities with actually occurring flows, the dyadic Relative Acceptance metric could be used, they argued, to identify interesting relations and segments of a trade flow network.

The Relative Acceptance metric was used as an indicator of dyadic prominence in several studies [e.g. 285, 275, 268; cf. 292]. A modification of the index was proposed by Goodman [283] where the null model of expected flows was based on the empirical topology: rather than assuming a null model distribution among all possible alters, Goodman’s alternative restricted null distributions to the existing topology. In the context of international relations, Brams found that, although the default Savage-Deutsch null model might be feasible in the context of international trade, the Goodman-variation and its assumption of a fixed topology was more useful when looking at the size and distribution of inter-country diplomatic missions [e.g. 268, p. 883, note 14].

While the Savage-Deutsch transformation has (to our knowledge) not been applied in blockmodeling, the deviational approach to valued blockmodeling suggested by Nordlund [295] shares the same principal objective of transforming valued ties with respect to the relative share of in- and outflows of each actor. Similar to the Relative Acceptance transformation, the deviational approach is based on a model of expected flows on the assumption of no interactions. However, instead of transforming (through iteration) each valued dyad into a singular value, the deviational approach analytically arrives at, for each valued network, two separate matrices capturing deviations from expected values for, respectively,

⁹The mathematical properties of Z-score-based iterative normalization is analyzed in [296], describing and exemplifying the procedure by beginning with Z-score-normalization on rows. Their example transformation is however done by starting with columns: if starting with rows instead, following their own description, the transformed values differ quite substantially from those obtained when starting the iterated normalizations with respect to columns.

¹⁰This model could be also used to compute the expected values of cell values, which could then be used for normalization (e.g. dividing the actual value with the expected).

outbound (RD) and inbound (CD) flows. The formulas are given and described in [295, p. 163].

As the deviational transformation produces two matrices rather than one, it is not immediately useful for conventional direct and indirect clustering methods. Instead, Nordlund [295] proposes both indirect and direct clustering approaches where both RD and CD are simultaneously used. Apart from measuring the degree of equivalence and identifying subsets of equivalent actors, heuristics for interpreting the resulting blocks based on both of these deviational matrices are also suggested. These will be addressed in subsequent sections.

The deviations for corresponding cells in RD and CD are in many cases quite similar. In the previous Germany-to-Austria trade example, this flow deviates positively for both, albeit somewhat differently: from Germany's perspective, this flow is 83 percent higher than expected ($rd_{DEU,AUT} = 0.825$), whereas this import flow to Austria is 120 percent higher than expected ($cd_{DEU,AUT} = 1.199$). In this example on intra-EU/EFTA trade (see below), there are however also examples of differences in the signs of the deviations. Whereas the trade flow from France to Germany is 13 percent higher than expected from France's point of view, it is 1.4 percent lower than expected from Germany's point of view.

6.4 Indirect clustering approaches

Capturing different specific notions of equivalence, indirect approaches and algorithms estimate the degree of such equivalences for each possible pair of actors in the network. Having established such measures of equivalence, suitable partitions of equivalent actors can then be identified using cluster analysis, such as hierarchical clustering, where the number of equivalent classes is either determined theoretically or guided by a suitable procedure for choosing an optimal number of clusters (see [290] for an overview of such procedures). Whereas this general procedure of indirect clustering methods is the same for binary and valued networks, below we focus on common indirect dis(similarities) applicable to valued networks – two for structural equivalence and the REGE algorithms for regular equivalence. We also discuss specific issues concerning valued networks when interpreting the resulting blockmodels derived from such indirect methods.

6.4.1 Structural equivalence: indirect metrics

For structural equivalence, defined as when two actors have the same relations (and lack thereof) with their alters [287, see], the operationalizations of indirect measures are quite intuitive. In a symmetric network represented as a sociomatrix, the row- or column-vectors for each pair of actors are compared with each other, whereas both the row- and column-vectors are compared for directional networks.¹¹

Although there are many measures of (dis)similarity that are useful for valued networks (e.g. Tanimoto, cosine, Manhattan etc.; [see 279, p. 181ff]), the most widely used indirect metrics of structural equivalence seem to be Pearson correlations and Euclidean distances. These indirect approaches thus transform the original sociomatrix into, respectively, ei-

¹¹For such comparisons of actors' profiles, there are different ways to cater for would-be self-ties [see, e.g., 307, p. 367]. Regardless of which metrics is used, it is important to pay special attention to self-ties (if present) and ties between the two units being evaluated for equivalence by using corrected measures [262]. E.g. in directed network with self-ties, if units i and j are to be structurally equivalent, a tie $i \rightarrow j$ should be equivalent to tie $j \rightarrow i$ and a self-tie of i to self-tie of j .

ther a correlation matrix (i.e. where a value approaching unity implies equivalence) or a distance matrix (i.e. where a value approaching zero implies equivalence).

Whereas both Pearson correlations and Euclidean distances are used to capture notions of the structural equivalence of actor-pairs in valued networks, they represent two slightly different takes on what structural equivalence means. Comparing the two metrics [e.g. 307, p. 374], the Euclidean distance (also called “social distance” [see 273, p. 95]) metric is sensitive to differences in means and variances of the tie values of two actors [280], while the correlation metric caters for such differences. From this, the correlation-based indirect measure is oriented to identifying similar profiles of alter ties [e.g. 307, p. 375], with the Euclidean metric of structural equivalence instead capturing similarities with respect to tie strengths. This is exemplified in [295] where blockmodels stemming from, respectively, the correlation- and Euclidean-based indirect metrics are compared for Baker’s citation data [257]. Whereas the Euclidean-based metric of structural equivalence produces clusters of actors with similar gross (in- plus out-) degrees, the clusters arising from the correlation-based indirect metric contained actors with wider variations in (unequal) relational capacities.

6.4.2 The CONCOR algorithm

Building on the above indirect measure of structural equivalence, the CONCOR algorithm is an iterative algorithm that builds on the phenomenon of the convergence of iterated correlations. As an independent co-discovery of Breiger and Schwartz [300, 270], the phenomenon of convergence of iterated correlations had previously been observed by McQuitty [289].

Starting with the correlation-based indirect measure of structural equivalence (see above), the CONCOR algorithm repeats correlating the correlation matrix. With few exceptions, this procedure converges to a matrix that contains only positive and negative unity, these corresponding to partitioning the network into two, arguably structurally equivalent, clusters. The CONCOR algorithm can then be applied to each of these subset, further partitioning the network in a hierarchical fashion.

As the CONCOR algorithm is based on correlations, it works for both binary and valued networks. This implies it is sensitive to differences in the means and variances of the valued relations of actors, i.e. there is an emphasis on similarities in the patterns rather than the strengths of ties. However, although the CONCOR algorithm has seen extensive use [see 270, 293, 304, see 286, for an extensive example], it has some well-known problems. First, CONCOR always produces a dual partition. For networks containing three very accentuated and similar clusters, the CONCOR algorithm always yields a split into two parts. Second, the choice of clusters for subsequent CONCOR splits can be arbitrary. In [304], the choice of which clusters to split was based on the size of the remaining clusters, implicitly assuming that the sets of structurally equivalent clusters would be similar in size. Third, the CONCOR algorithm can be quite sensitive to small variations in the input data. This was demonstrated in Breiger’s study [269] of OECD trade data: applying the CONCOR algorithm to both raw trade data and means-based iterative row-column normalized data (see the previous section), he found significant results in the partitions suggested by and derived from the CONCOR algorithm.

6.4.3 Deviational structural equivalence: indirect approach

Whereas the indirect approaches described above are applied to individual¹² matrices, these matrices can, as shown in the previous section, be subject to various types of transformations prior to clustering. However, as the deviational transformation yields two transformed versions of the original valued network, the standard indirect methods do not apply. Adapting the standard correlation-based formula to cater for both the RD and CD deviation matrices (see 3.5 above), Nordlund [295] proposed an indirect measure of deviation structural equivalence, where row- and column vectors were analyzed by looking at, respectively, the row-based (RD) and column-based (CD) deviations [295, p. 165].

Once calculated for each pair of actors, this indirect measure of deviation structural equivalence is then subject to hierarchical clustering or a similar method for finding equivalent actors. Both RD and CD are used in the subsequent interpretation (i.e. the identification of complete and null blocks) of the resulting blockmodel – see below.

6.4.4 Regular equivalence: the REGE algorithms

While structural equivalence implies having the same patterns of ties to individual alter actors, regular equivalence implies having the same patterns of ties to actors that are themselves equivalent actors (White and Reitz 1983). Unlike how structural blockmodeling examines a blockmodel for complete (filled with ties) and null (void of ties) ideal blocks, regular blockmodeling replaces the former with ideal regular blocks, meaning blocks where each row and column, respectively, contain at least one tie. Formulated by Sailer [298], regular equivalence was further developed by White and Reitz [308]. White and Reitz [308] also proposed the REGE algorithm as an indirect method for measuring regular equivalence between pairs of actors. Through an iterative point-scoring procedure, the algorithm measures how well the relations of each pair of actors match each other in terms of both the strength and equivalence of the actors on the other end [308; 307, p. 479ff; 267, 265, 312]. A limitation of the REGE algorithms is that they only search for *maximal* regular equivalence¹³.

The notion of regular equivalence and the definition of a regular block is specified for binary networks. For formal definitions for valued networks, see [312]. Although the REGE algorithm applies to both binary and valued data, it has been shown to have some shortcomings, particularly when it comes to valued networks. Due to the workings of the matching function in REGE, a few similar ties of large magnitudes could very well dwarf several similar ties of lower magnitudes [see 267, 265]. The REGE algorithm thus tends to emphasize the strengths, rather than the patterns, of ties when estimating the degree of regular equivalence between pairs of actors. As noted by Žibera [312], more than one variant of the REGE algorithm exists.

¹²With the caveat of multiplex networks: the classical indirect measures of equivalence presented here also have implementations for networks with multiple layers of relations. For such situations, the resulting measures of equivalence thus represent a composite measure of equivalence for each layer. Examples of such applications are the classical series of world-system studies [304, 293, 303, 288], where multiple types of international relations and commodity classes were simultaneously analyzed when identifying structurally and regularly equivalent countries.

¹³Maximal regular equivalence is the regular equivalence where the classes are the largest possible. For binary networks, this also means that REGE algorithms are appropriate for undirected connected networks (no isolates), as then all units represent one regular equivalence class.

6.4.5 Indirect approaches: finding clusters, interpreting blocks

As the above-mentioned indirect methods result in dyadic measures of equivalence, the definition and interpretation of equivalence depend on the chosen metric and would-be pre-cluster transformations. The partition of the correlation and distance matrices derived from, respectively, correlation- and Euclidean-based indirect measures also depend on the chosen clustering method. Although agglomerative hierarchical clustering is often used to delineate clusters of equivalent actors, the choice of the agglomeration method – i.e. single-link, complete-link, weighted/unweighted average, Ward etc. – also affects the resulting partition. In addition, the choice of selecting a suitable number of clusters remains, a choice that can either be informed by theoretical considerations, a review of different partitions, or procedures or indices for determining the number of clusters, or a combination of these. These issues concerning indirect-based partitioning apply equally to binary as well as valued networks and are thus not discussed here further.

Once a clustering and a corresponding blockmodel has been established, the subsequent step is typically to interpret the resulting blocks. For binary networks (including pre-cluster dichotomized valued networks), such interpretations are fairly straightforward, i.e. by comparing how the patterns of binary ties compare to the set of ideal blocks for structural, regular, or generalized equivalence. The comparison between valued empirical blocks and such ideal block types is however more complicated where valued networks are involved. Žiberna [310, p. 108] specified ideal blocks for valued networks, which though are only applicable in cases where raw values are compared (e.g. in terms of indirect approaches, if Euclidean distance-based matrices are used and not correlation or deviational based ones). When using homogeneity blockmodeling and structural equivalence, this means the blocks are represented and interpreted based on block means (or some other central values) of original and/or transformed data (if a transformation was used prior to blockmodeling). Using block means is also common and very suitable for Euclidean distance-based approaches (for valued or binary networks).

To match empirical valued blocks with the set of ideal binary blocks, post-cluster dichotomization can be employed. This can work well if the notion of equivalence includes magnitudinal differences of tie strengths, e.g. how the indirect Euclidean measure of structural equivalence (and perhaps also the REGE algorithm) works. If the chosen indirect method does take unequal relational capacities into account, e.g. such as the correlation-based metric, post-cluster dichotomization using a system-wide threshold of the raw valued data will emphasize the strength of ties when identifying various ideal block types. However, if a pre-cluster transformation for countering unequal relational capacities is applied, a post-cluster dichotomization of these transformed values could be used to identify prominent tie patterns for comparison with the ideal binary blocks.

To identify regular blocks in valued networks, Nordlund [294] proposes a post-cluster heuristic. Given a determined partition into regularly equivalent subsets, the raw valued data is normalized into, respectively, both a row- and column-normalized matrix (i.e. RN and CN above). Regular blocks are then identified by checking for row-regularity in the RN matrix and column-regularity in CN. Although determining tie prominence on the basis of both actors in a dyad, the heuristic involves dichotomization in this step as a standardized threshold is used to distinguish prominent and non-prominent ties.

Finally, for indirect deviational blockmodeling, Nordlund [295, p. 166] proposes that prominent ties be identified by looking at the signs and magnitude of deviations in both RD and CD. As the deviation for a valued tie may differ between RD and CD, i.e. depending on the actor in the dyad, the approach could result in valued ties being categorized as

neither prominent nor non-prominent, but non-determined. Density block images based on this approach could thus result in blocks where the densities are expressed as ranges rather than precise values.

6.5 Direct approaches

Direct approaches search for a partition based on criterion function that directly analyze network data, as opposed to indirect approaches that work on some (dis)similarity metrics computed from such raw data. Here we focus on generalized blockmodeling, especially the approaches for valued networks, including the deviational direct approach.

6.5.1 Generalized blockmodeling

Generalized blockmodeling (for binary networks) was already broadly discussed in the book *Generalized blockmodeling* [279] and numerous other papers [e.g. 259, 261, 262, 263, 264]. Generalized blockmodeling is a direct approach, meaning that a criterion function measures inconsistencies between an empirical partitioned network and one (or more) possible ideal blockmodel(s). In generalized blockmodeling, equivalence is defined by a set of allowed block types, which can either be set globally for the whole network or individually for each block. A block forms part of the network/matrix that represents ties from one cluster to another (these two can also be the same, i.e. blocks can also represent ties within a cluster). For each block, out of a set of allowed block types (which might also be only one), the block type with the minimum inconsistency vis-à-vis the empirical block in the same position is selected. The value of the criterion function for the whole network and a partition is then the sum of these block inconsistencies for all blocks.

The partition is optimized (by testing a large number of different partitions) so that the value of the criterion function is as low as possible. In the original as well as most other versions of generalized blockmodeling, relocation algorithms are used to optimize the criterion function [279, 310, 264, 315], although other attempts also exist [274, 272, 271]. For such relocation algorithms, the number of clusters must be set in advance.

There are several approaches to generalized blockmodeling. Each approach to generalized blockmodeling has its own logic concerning how the inconsistencies between empirical and ideal blocks are computed. Further, each also defines a set of possible allowed block types. For binary, valued and homogeneity blockmodeling (see below), these and their respective methods for calculating inconsistencies are presented in [310, Tables 2 and 3]. Implicit blockmodeling [260, 311, 313] is not discussed here as we find that other approaches are usually more appropriate, while approaches for signed networks [278] are discussed in Chapter 8 in this volume [306]. The deviational direct approach [295] is discussed in a later section. In this chapter, we only present the main ideas entailed in the selected blockmodeling approaches.

Binary blockmodeling only treats ties as either prominent (present) or non-prominent (not present). The inconsistencies are essentially computed as the number of times a tie is present where it should not be (based on ideal blocks) or vice versa. However, as ideal blocks are specified in terms of binary ties, the approach of computing the inconsistencies between the observed and ideal ties cannot directly apply to valued networks unless the valued ties have first been dichotomized appropriately. The question of whether it is appropriate to apply binary (generalized) blockmodeling to a valued network is thus often a

question of whether some dichotomization transformation is appropriate (see the previous section on transformations).

6.5.2 Generalized blockmodeling of valued networks

While valued blockmodeling is in essence similar to its binary counterpart in terms of “classifying” ties as either prominent or not, it also takes the value of the tie into account in such classifications. Therefore, the inconsistencies are computed as the sum of deviations from either 0 (when according to an ideal block a tie should not be present) or from a threshold we call m that determines, when a tie is considered prominent, if the tie should be present. Of course, when a tie should be present, values exceeding m do not cause inconsistencies. In the classical setting, a tie lower than $m/2$ is considered closer to being non-prominent and one higher than $m/2$ is considered closer to being prominent. Obviously, m is a parameter that must be selected by the researcher based on what can be considered a prominent tie. While some suggestions for how to select the value of the m parameter based on empirical data were given by Žiberna [310, 311], it is noted that valued blockmodeling is probably not appropriate when m cannot be selected based on what constitutes a prominent tie. Examples of how m was selected based on which ties we want to be treated as relevant can be found in the example section in the EIES friendship data and (after normalization) in the EFTA trade data.

Homogeneity blockmodeling is quite different because it does not “separate” ties into prominent or non-prominent, but simply strives to have selected values be as homogenous as possible. These selected values could be all tie values in a block (as in the case of complete blocks), values of some function over rows or columns (as in the case of f -(row/column-) regular blocks) or some other set of values. Therefore, the inconsistencies are measured by some measure of variability. Sum of squared deviations from the mean and sum of absolute deviations from the median were suggested by Žiberna [310] and are used within this approach. The sum of squares approach is in case of structural equivalence very similar to using average within-block variance suggested by Borgatti and Everett [266] as measure of blockmodel fit. The main advantage of using a measure not normalized by the size of a block (sum of squares) compared to using the normalized one (variance) is that the “errors” in small and large blocks have equal weight. The advantage of the sum of squares approach over the absolute deviations one is that mean is taken as a representative value (which is usually desired), while the advantage of the absolute deviations approach is that it is less sensitive to extreme values. For that reason, the sum of squares approach is usually the “default” option, leaving the absolute deviations approach to be used when we wish to reduce the effect of more extreme values like in one case seen in the EFTA trade example. Yet they often produce similar results.

One of the primary advantages of homogeneity blockmodeling is that there is no need to specify any parameters in advance (like the m parameter or the dichotomization threshold), while a disadvantage is that the compatibility of some block types is questionable. Originally, the approach also suffered from the so-called null block problem, caused by the fact that the null block in homogeneity blockmodeling is technically speaking a special case of the complete block (i.e. where all tie values are “homogenous” to 0) and thus only “identified” in real networks if the null block was without error. Yet this problem was solved later [314] by so-called restricted blocks where the value from which the deviations are computed is restricted to be no less than some pre-specified value. These restricted blocks (complete, f -regular, etc.) can be used instead of the classical blocks where we would like to identify null blocks (including imperfect ones). The characterization of ideal blocks and

formulas for computing block inconsistencies for binary, valued and homogeneity block-modeling are presented in [314, Tables 1, 2, and 3].

6.5.3 Deviational generalized blockmodeling

Just like in generalized blockmodeling described above, deviational generalized blockmodeling [295] searches for an optimal blockmodel and ideal blocks that reduce a penalty function that contains the total number of inconsistencies between ideal and empirical blocks. Allowing for the same set of ideal binary blocks as in (binary) generalized blockmodeling, the differences lie in how the various penalty scores are calculated and how the ideal blocks are identified. Beginning with the deviational transformation resulting in the row- and column-based deviation matrices (RD and CD, see section 3.5), these two matrices are then dichotomized into binary versions (RB and CB). The default (recommended) dichotomization here is simply based on the sign of the deviations in RD and CD, but one can also apply either a one-sided (positive) or two-sided (positive and negative) threshold to find prominent and non-prominent ties. In the case of a two-sided threshold, non-determined ties can also appear [see 295, p. 164]. The penalties for different ideal blocks are calculated by looking simultaneously at both of these binary matrices (RB and CB) [see 295, p. 165]. Apart from the standard set of ideal blocks as found in generalized blockmodeling [e.g. 279, p. 187], complete and null blocks exist in both a strong and weak variety, depending on whether the criteria for the respective block is fulfilled in either, or both, deviation matrices.

Since deviational blockmodeling allows for ties to be neither prominent nor prominent, but also non-determined/contradictory, the approach also provides measures of interpretational uncertainty for each ideal block type [see 295, p. 165]. Given that such ties could mean that the sign of the deviation in RD and CD differs, these uncertainties are thus kept out from the actual penalty function. For generalized deviational blockmodeling, it is suggested to use the sign of deviations when converting from RD/CD to RB/CB, find the optimal blockmodel(s), and then increase a two-sided deviation threshold until the total uncertainty score for the blockmodel increases. We can then state that a particular blockmodel is certain up to a specific two-sided deviation threshold, thus turning the in-bound cutoff parameter into an outbound indicator of overall interpretational certainty for the blockmodel [295, p. 164].

6.6 On the selection of suitable approaches

No study is the same – research questions, notions of equivalence, relevant aspects, and, of course, datasets vary between studies, making it difficult to provide the analyst with analytical schematics and rules of universal relevance. With the particular dilemmas pertaining to valued networks and the plethora of transformations and metrics available, this is even more the case with blockmodeling and clustering of valued networks. That said, we offer some guidelines we believe are generally applicable.

Whereas indirect approaches aim to capture notions of equivalence independently of how the resulting blockmodels appear, direct approaches identify suitable partitions and clusters on the patterns of ties of the final product, i.e. the blockmodel. Although effective search algorithms exist for finding partitions with low penalty scores, direct approaches are typically very time-consuming [279, p. 134], especially for large networks. Although such direct search algorithms should be run multiple times using different random starting par-

titions, they could end up with less-than-optimal solutions, i.e. where the criteria/penalty function reaches a local minimum. This problem is confounded the bigger the network is. Accordingly, direct approaches are mostly useful for relatively small networks while the indirect methods are better suited to dealing with large networks. What constitutes a small network and what a large network of course depends on computing power. We thus suggest using direct approaches when they can be estimated within a reasonable time. However, if we want the result to match a certain pre-specified blockmodel (e.g. hierarchical model, cohesive groups, core-periphery), generalized blockmodeling must be used (unless specialized approaches exist).

For blockmodeling valued networks, the choice between clustering methods and possible transformations must be based on the interpretation of the tie strengths and actors. If tie values have an equal interpretation across actors, such as when measuring playtime among pupils during a 45-minute lunch break, our general recommendation is to use the direct approaches for valued networks or distance-based measures (e.g. Euclidean) together with indirect approaches.

However, when actors seem to have unequal relational capacities, for instance as revealed by examining the distributions of tie values and/or valued degrees, another question must be addressed: should such relational capacities influence the result or not? If we perceive such differences as part of what defines equivalence, we once again recommend classical direct approaches and distance-based indirect measures applied directly to the raw valued data. If we wish to discount for such unequal relational capacities, we recommend the deviational direct approach and the correlation-based indirect measure – or the application of a suitable transformation as described above prior to applying a more classical direct or indirect approach. For instance, in our EFTA trade example, we used iterative row-column marginal normalization and on this normalized network used “classical” direct approaches, more specifically valued and homogeneity blockmodeling.

To provide general guidance regarding the choice of suitable clustering approaches for valued networks, we propose the control chart shown in Figure 6.2, which comes with two caveats. First, the chart and its recommendations are only approximate and each decision should be taken after also considering other factors, at least the aim of blockmodeling in the specific case and the exact meaning of the data. Second, the figure in most cases does not provide an exact decision, e.g. the exact transformations are not specified and the choice between direct and indirect methods remains open. We have given guidelines on these and other aspects above or in particular sections of this chapter.

In the examples that follow, we test different approaches on two different valued networks. As is evident in these examples, two or more approaches can be equally “good”, all depending on whether their respective clustering methods and pre- and post-cluster transformations capture what we intend to capture in our analysis.

6.7 Examples

In this section, we analyze two example networks: data on EIES friendship in time period 2 [282] and intra-EU/EFTA commodity trade in 2010 [295]. Whereas the trade data contain actors with unequal relational capacities, the Likert scale of the EIES friendship data depicts a network in which the actors, at least in theory, have equal capacities for specifying their outbound tie values and where each value on the scale corresponds to a specified degree of acquaintance.

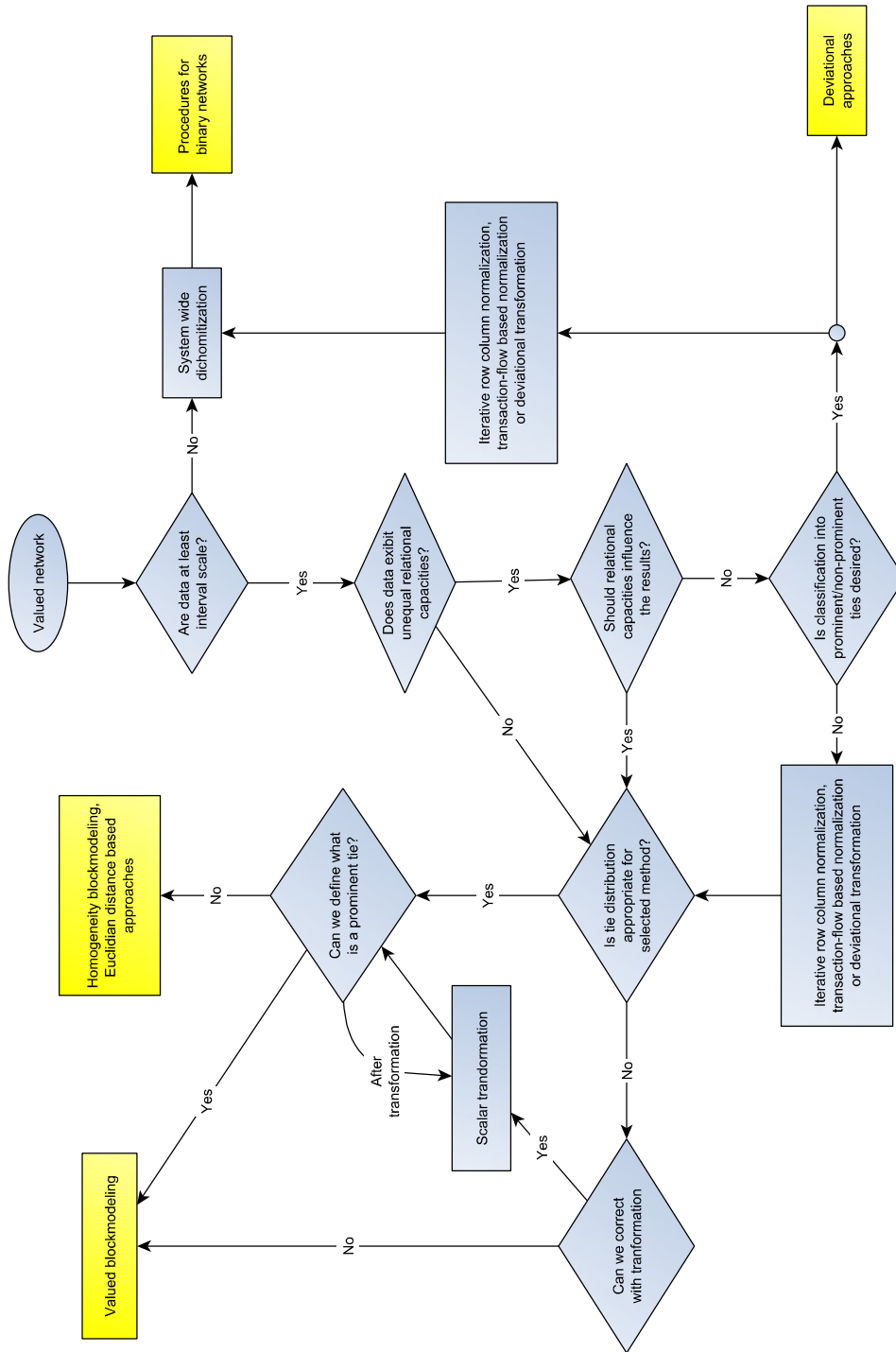


Figure 6.2: A "control chart" for choosing a suitable approach

We also exploit the simplicity of the first example (in the sense that no normalizations are used) to allow us to present its matrix partitioned into a larger number of clusters, on one hand using them to justify our selection of the number of clusters and, on the other, to allow readers to decide whether they agree with both our suggestions and the number of clusters based on scree plots.

6.7.1 EIES friendship data at time 2

The first dataset on which we demonstrate the use of valued blockmodeling techniques is Freeman's EIES¹⁴ acquaintance network at time point 2 [282]. The data were gathered in 1978 among researchers working on social network analysis. They were first introduced to a system for computer conferencing. Three networks were collected, two of which are acquaintance networks, one gathered at the time of introducing the system and one (which we are using here) after using the system for 7 months. In addition, the number of messages sent using the system was recorded (which we do not use). As mentioned, here we only use the data at the second time point for acquaintance data for 32 researchers. The acquaintance data were gathered using a survey where the participants answered using a 5-point scale coded as:

- 0 – a person who is unknown to me (or no reply),
- 1 – a person I've heard of, but not met,
- 2 – a person I've met,
- 3 – a friend,
- 4 – a close personal friend.

The Freemans [282, p. 367] warn that “[s]ince most of the participants were American speakers of English, it is difficult to determine whether their use of the term ‘friend’ refers to an affect-based tie. Americans tend to use that term to describe anyone from the most superficial acquaintance to a trusted lifelong intimate”. The coding is, of course, important for interpreting the results, but also for selecting various parameter values such as the threshold for dichotomizing networks (for use with approaches for binary networks) or the m parameter of valued generalized blockmodeling.

This may be considered a classical set of social science network data where the ties represent social relations that are usually measured with a questionnaire. It is also an example of a dataset where at least theoretically the relational capacities are equal for all respondents. Of course, one could argue that these do at least partly differ by respondents. However, these differences are typically not huge and, secondly, are not deterministically pre-determined (like in e.g. trade flow data where a small country simply cannot trade as much as a big country). Therefore, in such cases, the use of either a normalizing technique (or methods directly suitable for networks with different relational capacities) is neither needed nor desired.

Therefore, such data can be directly analyzed using methods for blockmodeling valued networks, i.e. without initial value transformations, and depending on the analyst's

¹⁴EIES data obtained from <https://sites.google.com/site/ucinetsoftware/datasets/freemanseiesdata>.

interest, the use of a network-wide dichotomization threshold could indeed be appropriate. One should note that, strictly speaking, the measurement scale used here is ordinal. However, as methods for blockmodeling valued networks assume at least an interval measurement scale, we treat these data here as interval scale data when applying generalized valued blockmodeling. Treating ordinal scales as an interval is often better than alternative solutions [316].

The classical approach to dealing with such data is to dichotomize the data. If we are explicitly interested in values above a specific threshold, e.g. for instance “friend” and above in the EIES data, dichotomization is indeed reasonable. Here the use of ties with a value equal to or greater than “friend” (3) seems natural, although other values might also be reasonable for certain purposes. Another reason for using this threshold here is that using a higher threshold (e.g. treating only the “close personal friend” values as ties) would be too stringent and result in just 51 ties, yielding a network density of 0.051 and an average in-out degree of 1.6.

For small (relatively dense) friendship networks, the structural equivalence model is appropriate¹⁵. Moreover, since the network is small, direct approaches are the most suitable¹⁶. Therefore, binary generalized blockmodeling according to structural equivalence was used on these data.

One question in need of answer is how to choose the number of clusters. We chose the number of clusters after reviewing the results for partitions into different numbers of clusters and after checking the scree diagram of criteria values at different numbers of clusters, all shown in Figure 6.3. Based on the scree diagram, four clusters seem appropriate (the curve “breaks” at 4, where subsequent additional clusters do not decrease the criterion function as much) and looking at the partitioned matrices confirms this result, as a five-cluster solution does not seem to increase the clarity of the block much. The four clusters that are obtained may be described as:

- Cluster 1: The most “popular” singleton cluster (i.e. containing a singular actor), with reciprocal ties to all actors in cluster 3.
- Cluster 2: A large cohesive group that also has relatively numerous ties (especially outgoing) to units from other groups.
- Cluster 3: A smaller cohesive group – with reciprocal ties to the most popular unit.
- Cluster 4: A peripheral cluster – the least internally connected cluster, mainly connected to the singleton cluster 1.

The number above the matrix plot indicates number of clusters. In matrices, the valued network is plotted. The ties that were converted to 0s in the dichotomization are represented by shaded cells, while the ones converted to 1s are represented by shaded completely filled cells.

Binary blockmodeling applies if we are strictly interested in ties above a certain level, e.g. “friends or more” above. Yet if we are more interested in acquaintanceship in general, we could use valued blockmodeling on the assumption that the data are at least on an interval scale (i.e. where each step between Likert-scale responses corresponds to the

¹⁵In addition, in this chapter we focus on issues connected to using valued data and therefore, as a rule, are sticking to the simplest equivalence that makes sense – usually structural equivalence.

¹⁶In general, direct approaches are suggested unless the size of network is too large to be analyzed in a reasonable time with these approaches.

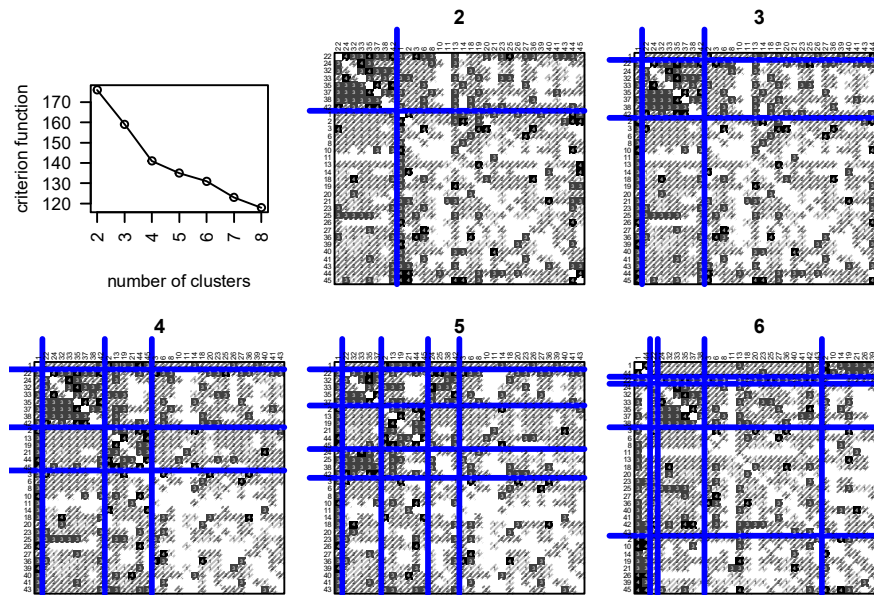


Figure 6.3: Binary generalized blockmodeling

same degree or depth of acquaintanceship, for this purpose, the m parameter must be set. On one hand, m should be set to a value that means the two units are strongly connected, or at the maximal range of the scale. Both of these rules would suggest 4 is a suitable value. However, a more precise (and perhaps a complicated) rule is that m should be set so that tie values closer to m than to 0 can be considered prominent, which compared to binary blockmodeling suggests using double the threshold that is used for dichotomization. This rule leads to the selection of $m = 5$, which is a more appropriate value although it means no tie can achieve this criterion¹⁷ [310]. However, the advantage is that ties 3 and 4 (i.e. “a friend” and “a close personal friend”) will be considered prominent and ties 0, 1, and 2 (i.e. a person they have only met or even less) will be considered non-prominent.

The results are presented in a similar fashion as for the binary blockmodeling in Figure 6.4. In this case, the scree diagram suggests three clusters (or at least eight) and, based on the review of the matrices partitioned into several clusters, this seems reasonable. However, after also inspecting the image matrices (Figure 6.5) we would opt for four or five clusters. We usually omit the presentation of the image matrices to save space, yet here they were the reason that four or five clusters are suggested. While in the threecluster solution it seems that no clusters are connected, the image matrices for the four- and five-cluster partitions show the first cluster (the single-most connected individual) has ties to cluster 3 and from all clusters but cluster 2 (and itself). Here we focus on four clusters for comparability with the results of the binary blockmodeling, although the five-cluster solution does find a new small cohesive group from the units from the peripheral cluster. In the case of four clusters,

¹⁷It also follows from this that complete blocks will have an inconsistency of at least the number of ties in the block.

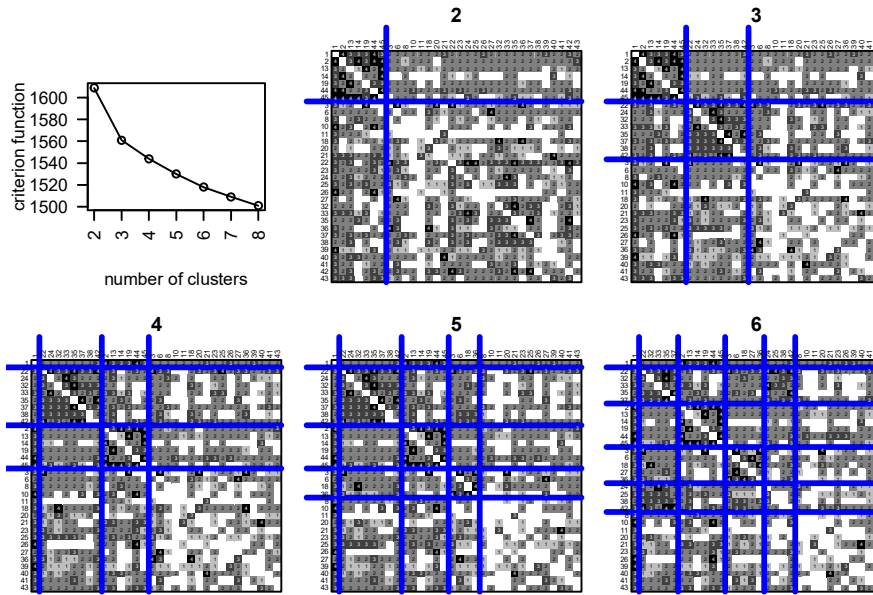


Figure 6.4: Valued generalized blockmodeling with $m = 5$. The number above the matrix plot indicates the number of clusters.

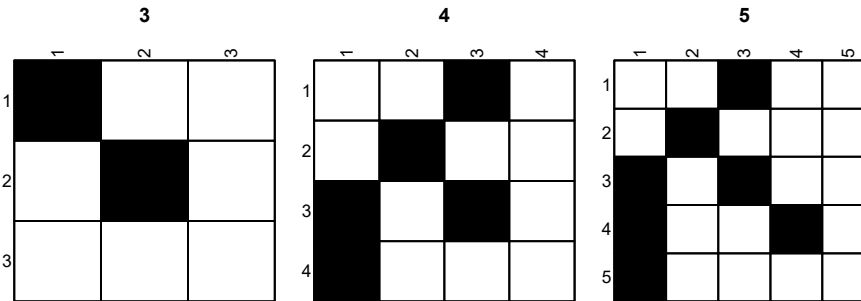


Figure 6.5: Image matrices for valued generalized blockmodeling with $m = 5$. The black squares indicate complete blocks and the white squares indicate null blocks. The number above the matrix plot indicates the number of clusters.

the interpretation of the clusters remains the same as for the binary blockmodeling. The membership of clusters is also very similar, only two units from a smaller cohesive group and a peripheral cluster change places (units 14 and 21). Unit 14 has all its prominent ties to or from the small cohesive cluster valued at 4 and most of its binary non-ties valued at 2 (almost relevant), which causes it to move into this cluster. On the other hand, unit 21 has all its binary ties to or from this cluster valued at 3, which was not strong enough evidence in the valued case.

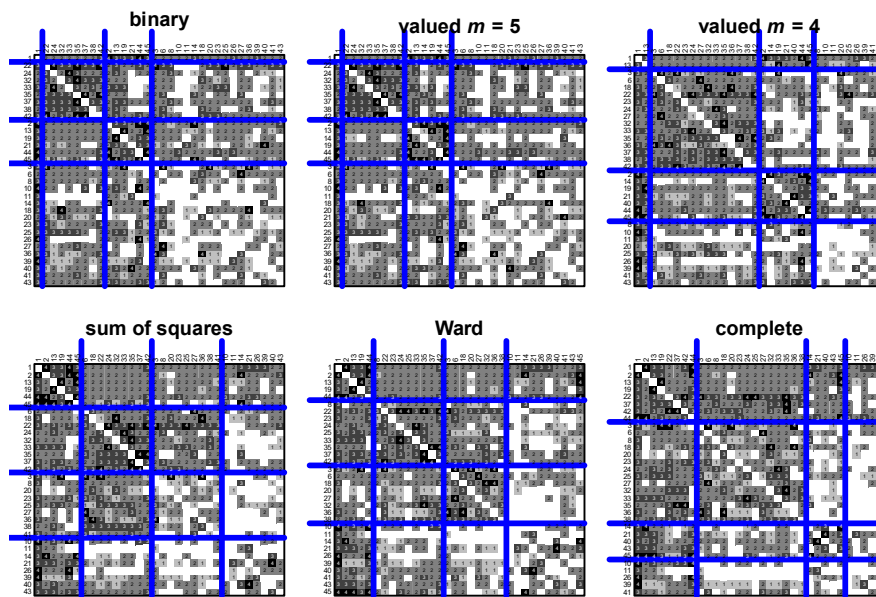


Figure 6.6: Four-cluster partitions by selected approaches

Other approaches were also tested on this dataset. These included the indirect approach using corrected Euclidian distance [262] with $p = 2$ with Ward and complete hierarchical clustering and sum of squares homogeneity blockmodeling according to structural equivalence [310]. As the correlation measure and various transformations (including deviation based) are theoretically unsuitable, we do not present their results here. The four-cluster results for these selected methods together with the selected partitions that were previously presented are shown together in Figure 6.6.

The first partition was obtained using binary blockmodeling and is the same as in Figure 6.3, but here it is drawn on the valued network without any modifications, showing that the possible impression of clarity (based on Figure 6.3) might be false. Comparing this partition to the valued blockmodeling one with $m = 5$ (the next figure), we see they are indeed very similar. This is expected because in both approaches values 0, 1, and 2 fit better in the null block and values 3 and 4 in the complete block. The only difference between these partitions is seen in the binary partition, where unit 14 is in the fourth cluster, while unit 21 is in the third, while these units are exchanged in the valued $m = 5$ partition. The reason for the difference should be searched for in the diagonal block of the third cluster, which in both solutions is classified as complete. Unit 21 has five 3s and four 2s as ties to or from other units of the third cluster. This makes it indifferent to these ties being in the null (fourth cluster) or complete (third cluster) block in both approaches. On the other hand, unit 14 has four 4s, five 2s and one unity tie as ties to and from units from the second cluster. Given that there are more ties below 2.5 than above, this fits better than the null block in the binary approach. However, the sum of distances to 5 is 23, which is less than the sum of distances to 0, which is 27. This makes these ties better fitted for the complete blocks in the valued $m = 5$ approach. The third figure also captures a valued blockmodeling partition, although here the value of $m = 4$ is used. This means that now value 2 (“a person

I've met") is as close to complete as it is to the null block. Consequently, complete blocks become larger.

In the bottom row, approaches based on sum of squares or (corrected) Euclidian distance are presented. All these tend to cluster units together with exactly the same tie values and thus the blocks in these partitions are much more homogenous. This is especially evident in the blocks from the first to the second and third cluster, which contain almost only 2s in these solutions (especially homogeneity sum of squares and Ward's solution). In particular, the results of sum of squares blockmodeling and Ward's hierarchical clustering are very similar, which is expected because both try to minimize sum of squared deviations from the mean, for which the homogeneity generalized blockmodeling approach is more successful. For example, based on this we can see that units of cluster 1 are friends (or more) with each other and some of them with selected units from cluster 4, who have met practically everyone. On the other hand, units from cluster 3 have very mixed relationships with each other, although they all have met or are even friends with people from clusters 1 and 2 and have often not even heard (and have at most met) people from cluster 4. The results of the complete linkage hierarchical clustering are included here mainly to demonstrate that the choice of the hierarchical clustering approach has a big effect on the final result.

To conclude, in this example, we find valued blockmodeling (with $m = 5$) the most appropriate method, although binary blockmodeling (with threshold 2.5) comes close behind. Both methods try to find groups that have either a friendly or non-existent relationship with each other, which we find more important than the homogeneity of the ties in this example. The advantage of valued blockmodeling is that it considers how far ties are from these "ideal" (friends, non-existent) states.

Sum of squares blockmodeling can, on the other hand, provide additional insight by trying to find as homogenous blocks as possible, which would be relevant if we were also interested in other levels of the continuum and not only in whether they are closer to being friends or not having a relationship.

6.7.2 Commodity trade within EU/EFTA 2010

This example consists of the total bilateral commodity trade between the 30 countries within EU and EFTA for the year 2010[295]. This network is almost a complete graph, having a topological density of 99.8 percent. Stated in millions of US dollars, tie values range from 1 to 103,434, with a mean of 3,954, a median of 483, and a standard deviation of 10,749.

If binary blockmodeling were to be used on this almost-complete graph, some kind of dichotomization would be necessary. On such a rich valued dataset, we think this should really be avoided.

As is almost a characteristic feature of international trade data, the actors (countries) in this network have vastly different relational capacities. Even if we assume that countries are equally integrated into the global economy, their economies are hugely different in magnitude, resulting in widely different perceptions of what constitutes an important trade flow for each country.

As seen in the previous section on transformations, there are several ways to take the values into account. One way is to simply use the raw data: as argued by Smith and White [303], the normalization of trade data could introduce artifacts for small flows, and the magnitudinal differences between countries constitute an important aspect of the trading system at large. While we agree with this, suitably transformed data (before or within

the clustering procedure) can reveal different aspects of the data. In this example, we demonstrate which kinds of results and insights can be obtained by following both paths and explore several options along both paths. However, in all cases we focus only on structural equivalence. As our emphasis here is on transformations, not many clustering techniques will be applied. We begin by analyzing the raw data.

6.7.2.1 Analyzing raw and scaled data Since there are no theoretically given threshold for values to be considered as relevant, homogeneity generalized blockmodeling or equivalent direct approaches are the most appropriate. Due to the, in principle, superiority of direct approaches, we concentrate more on them. Therefore, we first use both homogeneity approaches, that is sum of squares blockmodeling and absolute deviations blockmodeling, on the raw data.

Figure 6.7 shows the results of the sum of squares approach. Based on the scree diagram and a review of several solutions, we deem the four-cluster solution the most appropriate. However, regardless of the number of clusters a core-periphery structure is revealed whereby the core countries are those with larger trade. In the four-cluster solution, the clusters are as follows, named using the traditional trimodality used in world-system analysis:

- Dominant : Germany, having the strongest ties to practically all countries, especially to those in the other core cluster.
- Core: France, Netherlands, Great Britain, Italy, Belgium. These are strongly linked to Germany and with each other. The Semi-periphery also trades with them.
- Semi-periphery: Spain, Switzerland, Austria, Poland, Sweden, Czech Republic. These have very weak ties with each other, somewhat stronger ties to the Core and the strongest ties to Germany.
- Periphery: all remaining countries. Very weak ties to most countries, with some exceptions (especially with Germany).

As these results and especially Figure 6.8 show, the cluster membership is practically determined by the combined trade volume (the valued degree is computed as the sum of all incoming and outgoing ties).

One could rightly argue the clustering obtained above is heavily influenced by the different relational capacities of the countries. This could indeed constitute the main aspect of the structural equivalence we wish to capture, and the above methods seem adequate for capturing such differences. However, the relative influence of such differences when determining structurally equivalent actors can be reduced by applying a suitable scaling transformation. This is what Mahutga [288] did with his trade flow data when he applied a log-transformation prior to clustering. For our EU/EFTA trade flow example, Figure 6.9 shows that log-transformations indeed bring the distribution of the tie values much closer to a normal distribution. The requirement for this transformation is that all values are positive. In our example, we have two exceptions to this because both trade flows between Iceland and Cyprus are 0. To avoid the problem of an undefined logarithm, we added 1 to all tie values. If these values were to be lower than 1, the logarithm would be negative, although this does not pose a problem for homogeneity and similar approaches that can handle negative tie values. The smaller this value compared to what the smallest non-zero value in the network will be (in our case, we made it the same as the smallest non-zero value), the bigger the distinction between the zero ties and the other (non-zero) tie values.

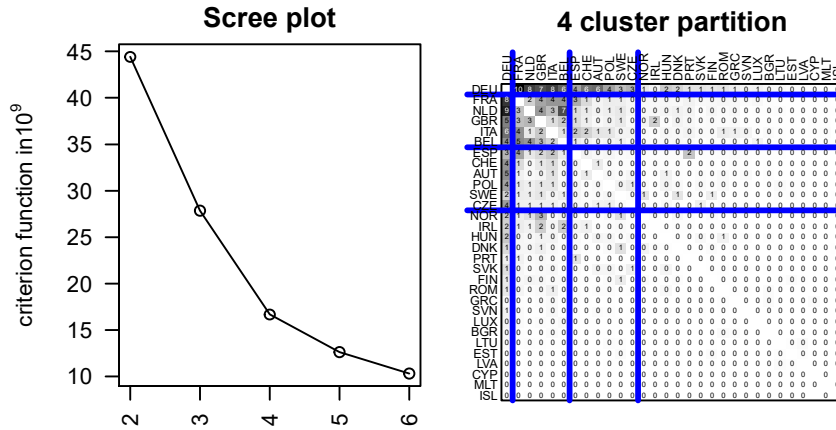


Figure 6.7: Results of sum of squares blockmodeling

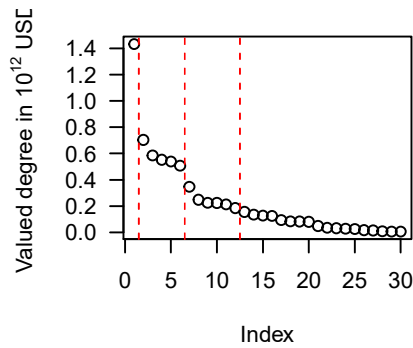


Figure 6.8: Valued gross degree (in- plus outdegree) of countries ordered by a four-cluster sum of squares solution. Dotted lines indicate cluster borders. The results are very similar if we use absolute deviations blockmodeling, although the size of some clusters changes (mainly a few countries move “upwards” in the hierarchy).

The results of this analysis are presented in Figure 6.10. We can see that the general core-periphery structure remains, only Germany loses its special cluster and the core groups generally become larger. Consequently, judging by the scree plot, a three-cluster partition now seems the most appropriate (i.e. without a singleton Germany cluster).

6.7.2.2 Analyzing iteratively row-column normalized data To counter the unequal relational capacities within this network, we first used iterated row-column marginal normalization. Here the results show strong evidence of trade gravity effects, i.e. where the magnitude of trade between countries is inversely related to the spatial distance between them. As this normalization is prone to producing relatively extreme values for countries trading small volumes, we use absolute deviations blockmodeling on the transformed network. The results are presented in Figure 6.11 and Figure 6.12. Whereas the scree diagram (Figure 6.11) does not suggest a suitable number of clusters, we found the six-cluster parti-

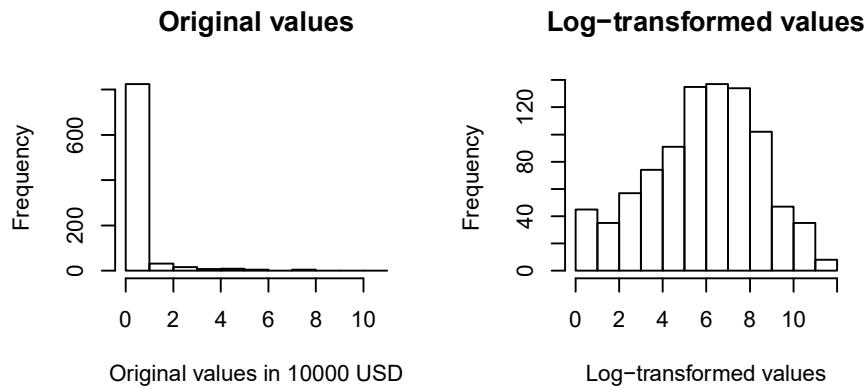


Figure 6.9: The distribution of the original and log-transformed tie values

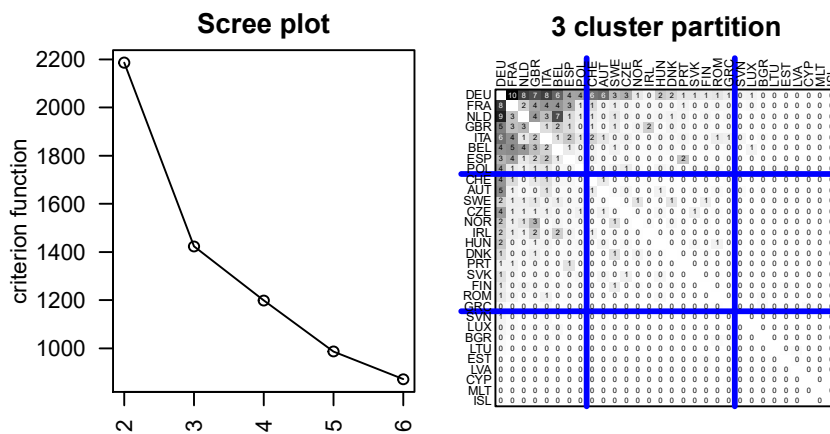


Figure 6.10: Results of sum of squares blockmodeling on the log-transformed network

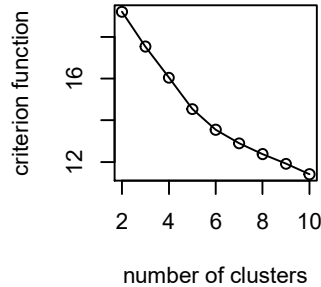


Figure 6.11: Scree diagram for the absolute deviations approach on an iterated row-column marginal normalized network

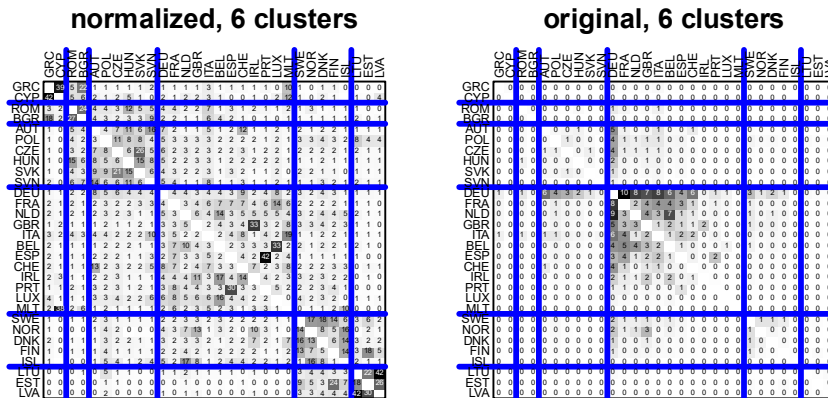


Figure 6.12: Six-cluster partition obtained using absolute deviations on an iterated row-column marginal-normalized network depicted on a normalized and original network

tion to be both substantially interesting and representing a suitable balance between details and aggregation.

In this solution, we have two clusters consisting of two neighboring countries, which are listed below the first. The remaining clusters represent specific sub-regions in Europe. These are:

- Greece and Cyprus: Greece is not exactly a neighbor of Cyprus, but it is its nearest neighbor in the dataset and, more importantly, the majority ethnic group on Cyprus is Greek.
- Romania and Bulgaria: The reason for them comprising a separate cluster is not only the strong ties between each other, but also that they are strongly connected to both the Greece/Cyprus cluster and Central-Eastern Europe.
- Central-Eastern Europe: Austria, Poland, Czech Republic, Hungary, Slovakia, and Slovenia.

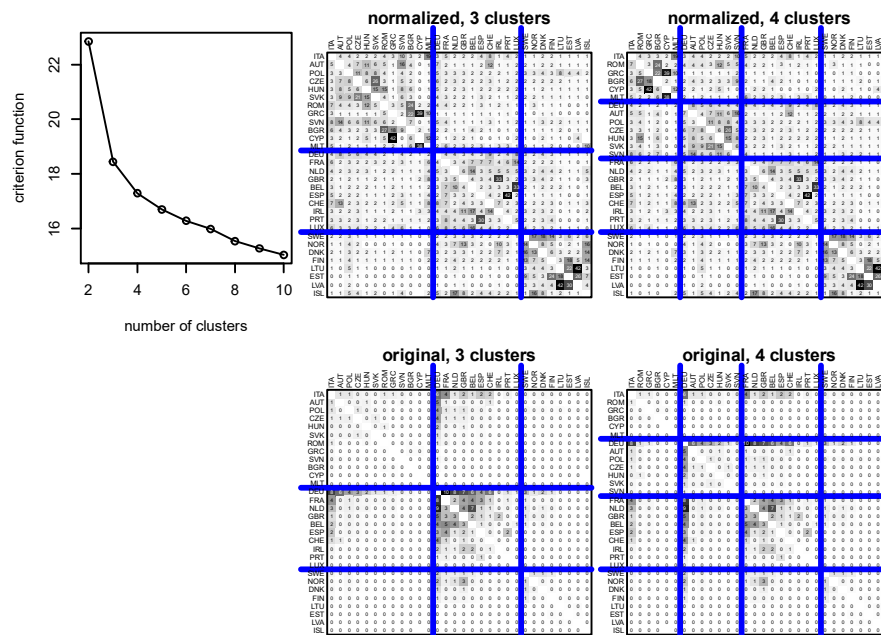


Figure 6.13: Valued blockmodeling results with $m = 2/29$ using an iterated row-column marginal normalized network depicted on normalized and original data (network)

- Central-Western Europe¹⁸ (countries not listed elsewhere).
- Northern Europe (Scandinavia): Sweden, Norway, Denmark, Finland, and Iceland.
- Baltic countries: Lithuania, Estonia, and Latvia.

On such an iteratively-normalized network, we could also specify which ties represent a relevant tie. Assuming a uniform distribution of tie values, the expected tie value here is $1/(\text{number of units}-1)$, which in our case is $1/29$. With this, the use of valued blockmodeling is appropriate where m is set to twice that value, which is $2/29$. The results are quite different; however, the regional trade-gravity effects are still obvious. The difference occurs as in the valued blockmodeling approach, values over m only cause inconsistencies in the null blocks and do not increase the fit in the complete blocks. The similarity of ties (apart for being close to 0 or m) is also not important. Fewer clusters are optimal in this case (three or four, with the former being more pronounced). The results for the three- and four-cluster solutions (on normalized and original networks) are presented in Figure 6.13. We selected the four-cluster solution due to its easier interpretability as the partition represent South-Eastern Europe, Central-Eastern Europe, Western Europe, and Northern Europe + Baltic.

¹⁸This cluster is split into more “seafaring” and more inland-oriented countries in the eight-cluster solution.

6.7.2.3 Indirect approaches In this example, where possible we are focusing on the direct approaches. However, using indirect approaches brings the advantage that some distance measures can be used which do not have their equivalents in direct approaches. Two of such distance measures are:

- Corrected (Squared) Euclidian distance based on separate row and column (sum) normalization, where the distance is computed among rows for row-profiles and among columns for column-profiles.
- Using correlation (or actually $(1 - \text{correlation})/2$) as a distance measure.

The dendrogram for the first measure (Euclidian distance on separately row and column normalized networks) using Ward's hierarchical clustering together with the four-cluster solution¹⁹ on both the original and separate row and separate column marginal-normalized networks is presented in Figure 6.14. The clusters in this partition represent Central-Eastern Europe, Western Europe, Northern Europe (including the Baltics), and South-Eastern Europe (including Slovakia).

The results for the second measure (correlations), again using Ward's hierarchical clustering, are shown in Figure 6.15. This procedure also captures trade gravity effects and regional trade concentrations. The five-cluster solution produces the following clusters:

- Western Europe,
- Central-Eastern Europe,
- Northern Europe and Ireland,
- Greece, Cyprus, and Malta,
- Baltic.

Using these indirect approaches admittedly has a certain appeal over using either generalized blockmodeling on iteratively row-column marginal-normalized networks or using a deviational approach: the transformations here are much simpler or not required at all (e.g. when using the correlation-based indirect metric), thus making it easier to motivate their use. Of course, generalized blockmodeling's main advantage is its versatility, particularly its ability to use other equivalences/block types and to use pre-specified blockmodels.

The final approach we try here is the indirect measure of deviational structural equivalence. This dataset was initially analyzed to demonstrate deviational blockmodeling [295], including its indirect approach. We use the same measures of deviational structural equivalence (correlations on RD and CD matrices) as suggested in [295] but apply Ward-type hierarchical clustering instead of the weighted-average version used in [295, p. 174]. In addition, rather than using an incremental two-sided deviational threshold (post-clustering), we identify prominent and non-prominent ties based on sign similarity in RD and CD. Here, non-determined ties are thus those where the corresponding values in RD and CD have different signs. The four-cluster partition is given in Figure 6.16 below.

Despite using the same deviational coefficients as those in [295, p. 174], the partition we obtain in Figure 6.16 differs somewhat from that of the original study. The differences are due to the particular linkage-method used in each study: whereas Nordlund used

¹⁹Based on the dendrogram, the appropriate number of clusters is not very clear. The four-cluster solution is the last before singleton clusters emerge and is one of the more suitable ones based on the dendrogram.

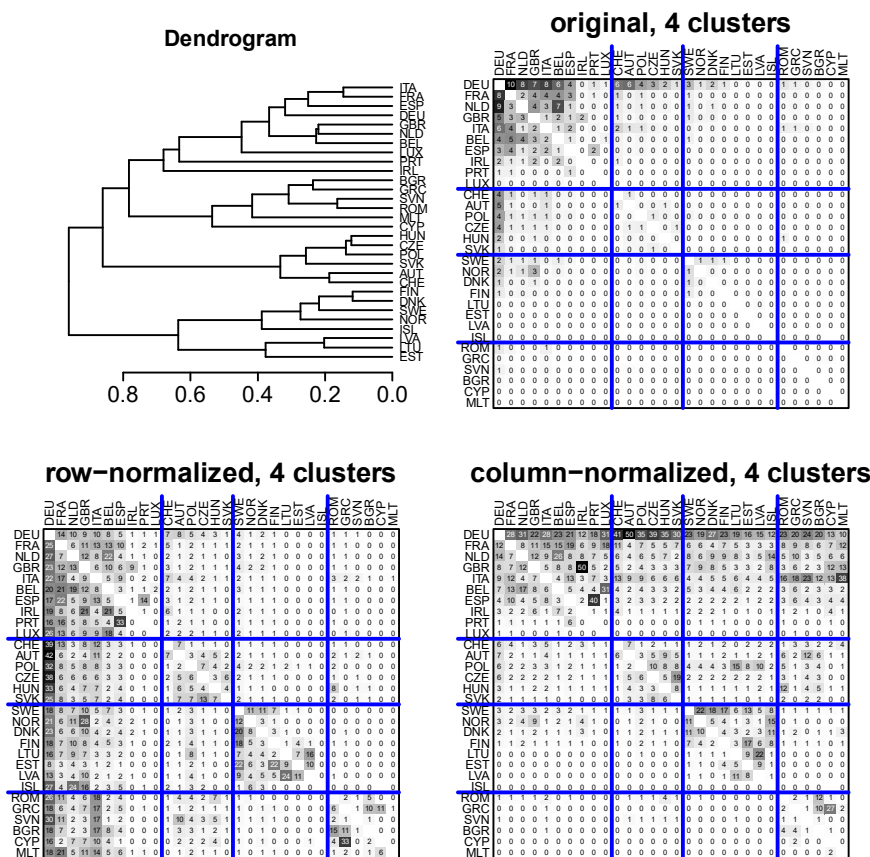


Figure 6.14: Dendrogram plus four-cluster solution for hierarchical clustering (using Ward's criteria) on Euclidian distance on separately row and column normalized networks depicted on original, row-normalized and column-normalized data.

weighted-average clustering (WPGMA), Figure 6.16 is obtained using Ward's hierarchical clustering. Thus, like for the case of binary networks (as well as any application of hierarchical clustering), the resulting partitions are not only due to the choice of picking a specific indirect measure of equivalence and possible preceding transformations, but the results also depend on the particular clustering methods used.

6.7.2.4 Summary This section demonstrated how trade flow data, in which units have vastly different relational capacities, can be viewed from at least two different angles. The first uses raw (non-normalized) or scaled networks whereby a core-periphery structure emerges, with partitions that are almost solely defined by the valued degree of the units – i.e. a strength-oriented focus. The other view is that based on somehow transformed data, where we try to cluster units not on the similarity of their raw ties but the ties where we somehow try to compensate for their differential relational capacity. When following this route, which we could call a pattern-oriented focus, a clear structure of cohesive groups

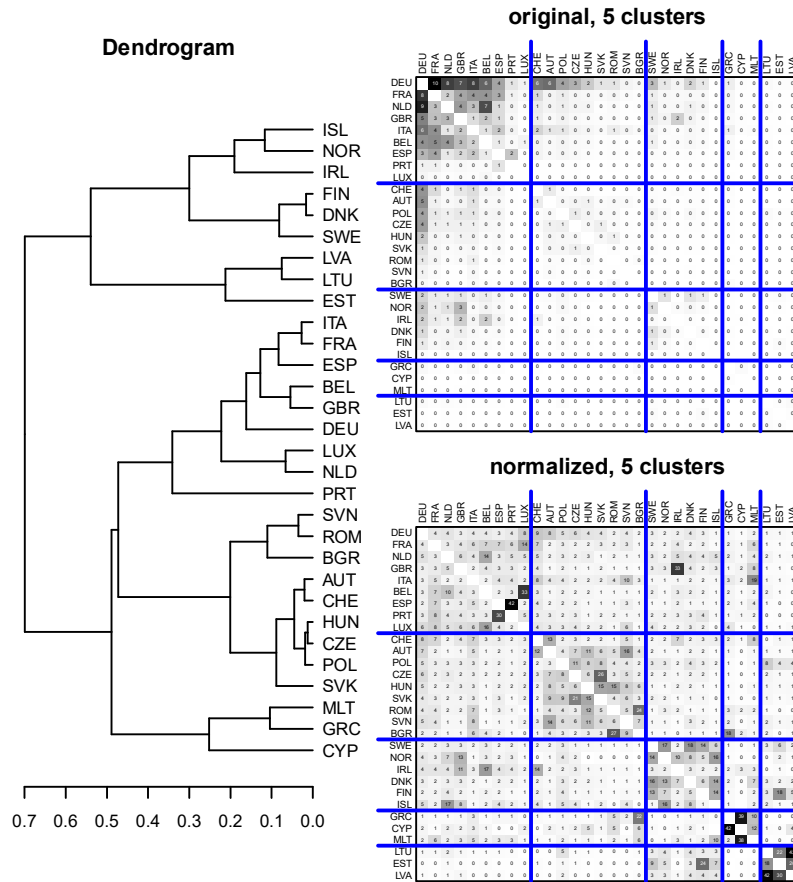


Figure 6.15: Dendrogram and the five-cluster solution for hierarchical clustering (using Ward criteria) on correlations on the original network (depicted on original and iteratively row-column normalized networks)

emerges, indicating a preference for regional trade. While both views may be relevant, given that the second one is not solely influenced by degrees, we could say that this one in some way better harnesses the richness of the relational data. Of course, much of this was already pointed out by Nordlund [295]. Yet, here we have shown that also other approaches (especially when using appropriate transformations) can be used to produce similar results as those emerging from the deviational approach.

Whereas this example primarily applied direct methods, we also approached the data using various indirect correlation-based measures of structural equivalence, testing different pre-clustering transformations. When discounting for unequal relational capacities, the indirect approaches also seem capable of capturing patterns of regional trade. However, while the indirect deviational measure seems adequate for capturing such effects, we demonstrate how the choice of the clustering heuristic can, at least in part, affect the obtained partitions. Thus, even though a transformation might assist in capturing struc-

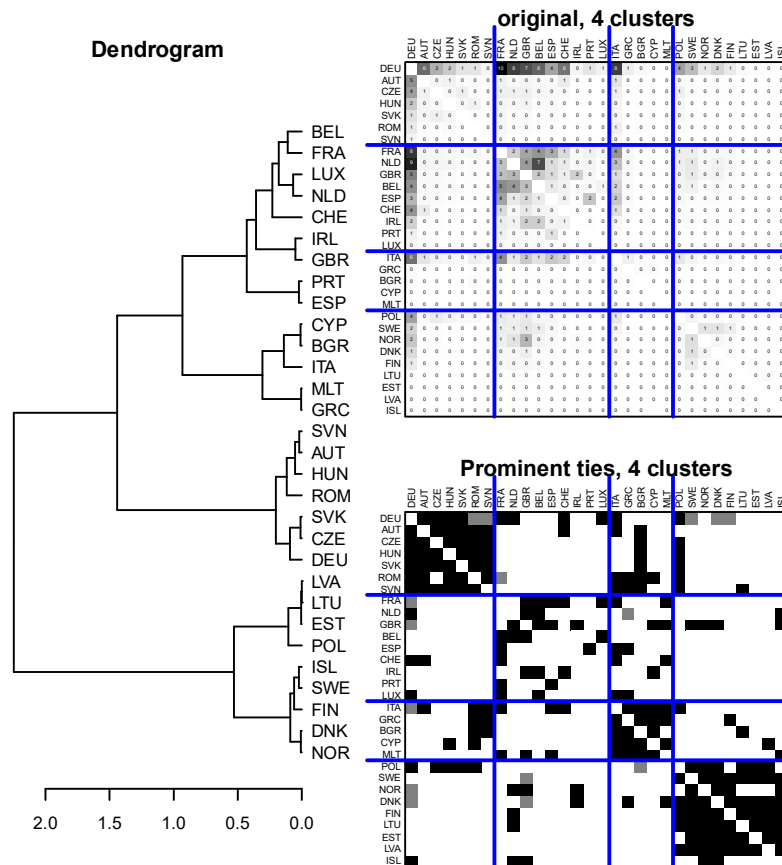


Figure 6.16: Dendrogram and four-cluster solution for Ward's hierarchical clustering on devianational correlations on the original network. Top blockmodel: original values; bottom blockmodel: prominent (black), non-prominent (white) and contradictory ties (gray).

tural equivalence that discounts for unequal relational capacities among actors, the indirect blockmodeling of valued networks is – similar to their binary siblings – equally influenced by the particular clustering heuristic we use.

6.8 Conclusion

As suggested in the multitude of available approaches, the clustering and blockmodeling of network data is anything but a trivial exercise. Not only do many different definitions of equivalence abound, but there are also multiple heuristics and approaches for identifying and measuring equivalences and the classification of actors according to shared notions of equivalence. Of the two broad approaches for doing this, i.e. direct and indirect, the former has evolved into a general analytical framework – generalized blockmodeling – that

allows different types of equivalences to be captured. The indirect approaches, constituting various approaches and algorithms for capturing pair-wise measures of different types of equivalences, typically imply additional choices and dilemmas when it comes to the actual clustering of such pair-wise indirect measures.

Whereas the above applies to the clustering and blockmodeling of networks generally, the existence of valued ties implies additional dilemmas for the analyst. Indeed, providing richer, higher-resolution insights than their binary counterparts, the comparison of ties in a valued network is not necessarily as straightforward as is the case with binary networks. To this end, several possible clustering methods for valued data and a range of transformations can be applied when clustering valued networks. However, prior to this, the analyst has to understand how the different transformations and clustering heuristics, direct and indirect, carry different ideas of equivalence that stretch beyond their binary counterparts. The core question that must be addressed when clustering valued networks is what the tie values actually mean in the context of equivalence. Are the tie values directly comparable across the network, or do actors have different capacities when it comes to tie values? Is the sought-after notion of equivalence related to such differences or should such unequal relational capacities be discounted for?

This chapter has addressed various clustering approaches that apply to valued networks. Transformations play an important role here in both preparing valued networks for use with existing direct and indirect approaches as well as interpreting the resulting blockmodels. The most rudimentary of these is the dichotomization of valued data into binary data, an approach we find inherently problematic for multiple reasons. Even if we assume equal relational capacities among actors and are able to set a system-wide threshold to distinguish prominent and non-prominent ties, it will inevitably reduce the resolution of the data.

Scaling transformations can be used to adjust the overall distribution of the tie values; however, they do not account for unequal relational capacities. Iterative row column normalization can be used to in a sense adjust for unequal relational capacities as it makes the in, out, and overall (valued) degrees of all actors equal. If classification into prominent and non-prominent ties is needed and, importantly, it makes sense in the data and research context, system-wide threshold dichotomization can be employed. Other transformations explicitly aim to identify prominent ties in valued networks where actors have unequal relational capacities. We examine transaction flow models and a novel deviational transformation, two transformations where the prominence of ties is determined based on the different relational capacities and degrees of actors in a dyad.

We also examine indirect measures of equivalence that are applicable to valued networks, exemplifying this by testing various approaches, transformations, and measures on two datasets – the EIES friendship network and commodity trade among countries in the EU/EFTA region. For structural equivalence, we note (as has previously been noted) that the two most common indirect measures – Euclidean distances and correlation-based – operationalize different takes on structural equivalence. Whereas the Euclidean distance-based indirect measure takes tie values into account in its measure of structural equivalence, the correlation-based metric is more inclined to capture the profiles, rather than the strengths, of the ties.

Something similar can be said for generalized blockmodeling approaches to valued networks [310] and the deviational blockmodeling of Nordlund [295]. While the former focuses on actual tie values, the latter concentrates on whether these values are lower or higher than expected (based on a no interactions model) and therefore patterns of ties. As such, these approaches offer different views on the network and thus complement each other.

For both direct and indirect approaches, the approaches that focus more on tie strengths can often offer similar analysis as those focusing on profiles/deviations, the data are suitably transformed prior to analysis. However, such transformation is not always trivial or even possible.

Our general recommendations are:

- One should avoid the dichotomization of valued networks (and application of binary approaches) whenever possible, unless the tie values cannot be considered to be measured on at least an approximately interval scale.
- The approach used should be selected while considering what the ties or tie values mean and the properties of the blockmodeling solution we are looking for:
 - If all units in the network have equal relational capacities (like friendship data), approaches focusing on tie strengths should be selected.
 - If all units do not have equal capacities, we must decide whether we want such differences in relational capacities to influence the results or not.
 - * If we do, once again approaches focusing on tie strengths should be selected.
 - * If we do not want different relational capacities to affect the results, then correlation or deviational based approaches should be selected. In some cases, using strength-focused approaches after suitable transformations is also appropriate, while in others finding suitable transformations is too problematic.
- If equivalent direct and indirect approaches exist, direct approaches are preferred long as the network is not too large to be analyzed by these approaches in a reasonable time. However, some indirect approaches (e.g. correlation-based ones) do not have an exact equivalent in direct approaches.
- When generalized equivalence or pre-specified blockmodels are needed, generalized blockmodeling approaches must be used.

While this chapter has aimed to shed some light on contemporary approaches for blockmodeling valued networks, several pertinent unsettled questions should be noted. One question is how the deviations of actual ties from the expected ones based on a model of no interactions should be measured. Second, whereas our focus in this chapter was on structural equivalence, we find it important to also explore how different transformations can be used in conjunction with other types of equivalences. Third, related to both of these, we argue the overarching question for blockmodeling of valued networks concerns the triadic interdependence between, respectively, pre-clustering transformations, notions of equivalence, and substantive interpretations. As each step in blockmodeling a valued network offers so many distinct choices – whether the valued data should be transformed, which transformation to use, different notions of equivalence, various methods to identify such, methods and choices related to the clustering of actors, etc. – these choices must be informed, guided, and motivated by the specific theoretical and substantive underpinnings at hand. Thus, although we propose a general guideline for choosing methods depending on the dataset and the research questions, the context is, as always, crucial.

Acknowledgments

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